THE MECHANICAL INTERPRETATION OF JOINTS[1]

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PART I

OUTLINE

I. THE JOINTS OF MINE FORK

In the course of field work in eastern Kentucky, in 1917, the writer observed a case of local jointing which gave him the clue to the following investigation.

On Mine Fork, a few hundred yards above the mouth of Lacy Creek, in Magoffin County, close to the Morgan and Johnson county lines, in a nearly vertical cliff of a strongly cross-bedded, coarse-grained sandstone forming the top of the Lee Group of the Pottsville series, the system of intersecting joints shown in the accompanying sketch (Fig. 1) is exposed along the roadside. Unfortunately the commercial work in which the writer was engaged at that time did not permit him to spend any more time in that vicinity than was necessary for a hasty survey of this exposure.

[1] Part I of this paper was presented, in essence, at the last meeting of Section "E" of the American Association for Advancement of Science, at St. Louis, December, 1919.

It was found that (*a*) The jointing is confined to the upper third (or even less) of the massive sandstone which is here about 100 feet thick. It is entirely lacking below. (*b*) It marks the crest of a minor anticline on the downthrow side of a conspicuous fault. (*c*) The average hade of the joint-traces on the practically vertical exposure which trends about in a NNE–SSW direction, is: set I: 27°—NNE; set II: 35°—SSW; inclosed angle: 62°. (*d*) The average strike of the joint-traces, measured on the horizontal surface of projecting ledges, is: set I: N 78 W; set II: N 27 W; inclosed angle: 51°. (*e*) The joints of set I are much better developed,

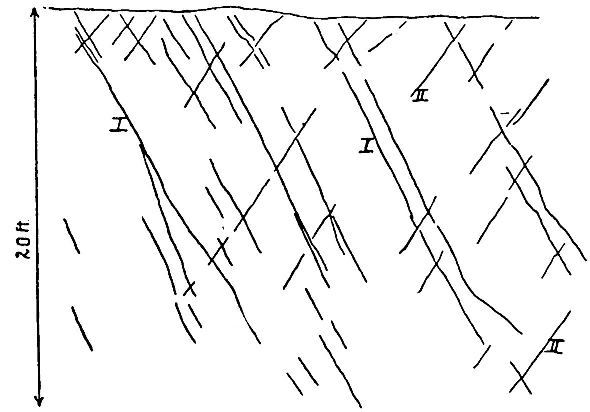


FIG. 1.—Jointing in vertical cliff of massive, cross-bedded sandstone on Mine Fork, Magoffin County, Kentucky.

longer, more continuous and more regular in their course than those of set II, both in the vertical and in the horizontal planes.

For two reasons the occurrence of this system of joints at this locality seemed surprising. There could be little doubt that these joints represented planes of shearing. The writer had, however, always associated the fracturing of hard materials by shearing with compressive stresses or, at best, with compound stresses resulting in torsion. But here he was dealing with a clear case of simple tension along the crest of an anticline, causing a hard sandstone to fail along typical planes of shearing.

He had also been accustomed to ascribe to the planes of maximum shear a general tendency to intersect at right angles. No

such tendency could be inferred from these joints, which intersect uniformly at an angle close to 60°, with the obtuse angle facing in the direction of the tensile stress.

II. HARTMANN'S LAW

Following the clue given by these observations, the author became acquainted with a book published in 1896 in Paris by L. Hartmann under the title *Distribution des déformations dans les métaux soumis á des efforts*,[1] containing a wealth of experimental data and a fascinating discussion of the lines forming on the surfaces of metals when strained beyond the elastic limit, known as Lüders' lines.[2]

When a highly polished plate of metal is subjected to a very gradually increasing simple tensile stress, the first permanent deformation is accompanied by the sudden appearance of one or several delicate straight lines or bands cutting in an oblique direction across its surface. Suitable illumination shows them to be depressions. When the stress is further increased, the existing lines widen and new ones appear, forming two conjugate systems of oblique lines, symmetrical to the direction of maximum stress and intersecting at a constant angle which in most metals (and rocks) is greater than 90°.[3] This angle remains unchanged with growing tension and is thus independent of the intensity of the stress. The final rupturing may entirely or partly follow these lines or cut across them at right angles to the tension.

Under compression, similar systems of lines form, but now the angle of intersection bisected by the direction of the compressive stress, for most rocks and metals, is smaller than 90°, and for the same material is the supplement of the one obtained under tension.

[1] Berger-Levrault, Paris, 1896.

[2] Called after Lüders of Magdeburg who first described them fully in 1860. "Über die Äusserung der Elastizität an stahlartigen Eisenstäben und über eine beim Biegen solcher Stäbe beobachtete Molekularbewegung," *Dingler's Polytech. Jour.*, Vol. CLV (1860), p. 18 (not seen).

[3] Ten good illustrations of strips of low steel showing yield lines developed under tensile stress, are given in H. Marten's *Handbook of Testing Materials* (translated by Gus. C. Henning), John Wiley & Sons, N.Y., 1899, Vol. I, Pl. 1, Figs. 3, 5, 12, 14–20.

The lines in this case are depressions only on one side, with the corresponding lines on the reverse side forming delicate ridges. Final rupturing, under compression, always follows these lines.

The great importance of these lines of Lüders for our purposes lies in the fact that they represent the outcrops of internal planes of yielding, differing largely in scale and degree of deformation, not in origin, from the planes of shearing observed on a large scale in nature.

In fact, in the small test piece as on a gigantic scale in nature, we see that the stress acts not uniformly on every unit of the mass undergoing deformation, but that it reaches a maximum along these geometrically distributed surfaces, while maintaining lower values in the volume between. We seem to be dealing here with a sharply defined application of the principle of least work.[1] At every point along every imaginary line of stress within a body undergoing elastic deformation there exists the tendency to shear in any one of an infinite number of directions all inclined to the direction of stress at the same angle, the sum of which forms two infinitely small cones joined by their apices.[2] Out of the infinite number of surfaces which may be obtained by connecting any two of such adjoining possible directions of shearing, those only will form which involve the expenditure of a minimum of energy.

When the lines of stress are not parallel, owing to the unequal distribution of stresses, the resulting surfaces of yielding may be very complex and the pattern of lines formed by their traces on the surface may be far from regular (Fig. 2*A*–*C*[3]).

In such cases Lüders' lines can be used to reconstruct the lines of maximum stress on any given test piece, by drawing the lines bisecting the angle of shear at every point of intersection of the shearing planes. Figure 2*C* represents the lines of stress derived in that way from Lüders' lines as obtained in the experiment illustrated in Figure 2*A* and *B*.

[1] H. von Helmholtz, "Über die physikalische Bedeutung des Princips der kleinsten Wirkung," *Wissenschaftliche Abhandlungen*, Vol. III, pp. 209–10.

[2] L. Hartmann, *op. cit.*, pp. 18–19.

[3] Hartman, *loc. cit.*, Figs. 48–50.

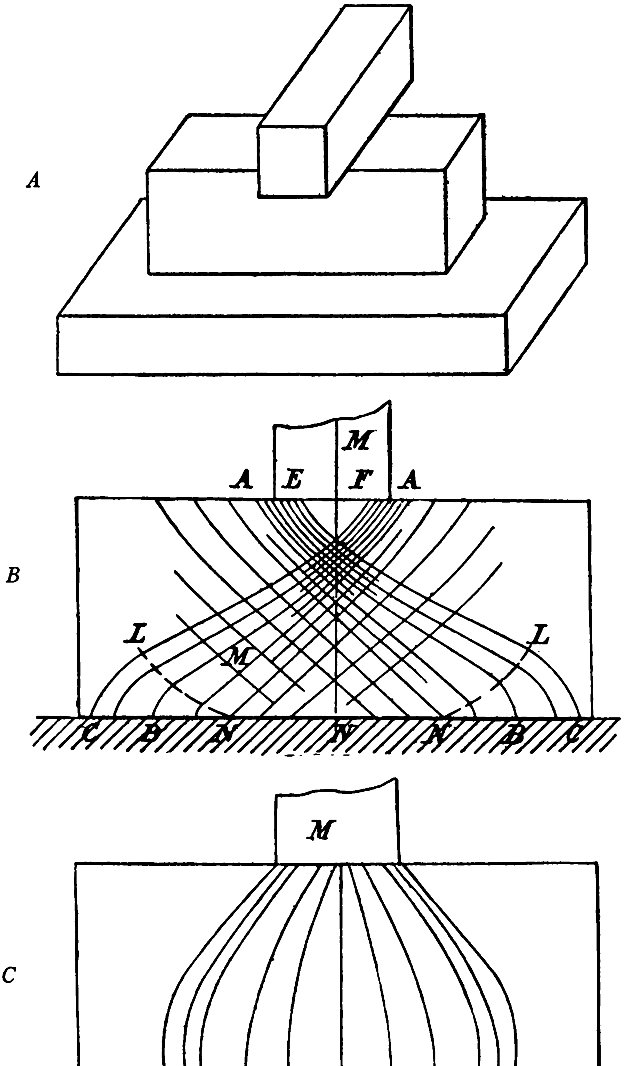


FIG. 2.—*A*. Arrangement used in one of Hartmann's experiments in which the test piece (in the center) was subjected to uniform compression over its whole base, while the upper surface suffered compression in the center only. (L. Hartmann, 1896.) *B*. Lüders' lines produced in the experiment illustrated in Fig. 2*A*. (L. Hartmann, 1896.) *C*. The theoretical lines of stress (bisecting the angles of Lüders' lines) reconstructed on the test piece illustrated in Fig. 2*B*. (L. Hartmann, 1896.)

The most irregular pattern of Lüders' lines results when the lines of stress are not parallel to the axis of the test piece, but intersect with it at varying angles. In that case, the angle formed by planes of yielding may be cut by the surface in all possible directions and the apparent angle of intersection of Lüders' lines as seen on the surface varies from point to point and must not be mistaken for the true constant angle bisected by the line of maximum stress of which it is only the oblique outcrop.

In 1900, O. Mohr published a mathematical study which led him to views practically identical with those of Hartmann. They may be summarized as follows:[1]

a) In all hard materials (except the most brittle ones), under tensional as well as compressional stresses, deformation by shearing takes place in two systems of intersecting planes of shearing.

b) Adjoining planes of one system are parallel.

c) The angle at which the two systems intersect is constant for any given material, that is, it is independent of the nature or intensity of the stresses involved.

d) For the same kind of material, this angle differs the more from 90° the harder and the more brittle the material is (e.g., hard or soft steel).

e) If we consider tension as negative compression, the law governing the arrangement of the yield planes with reference to the principal axes of stress which will be referred to as *Hartmann's Law*, can be expressed as follows: In brittle materials, *the acute angle formed by the shearing planes is bisected by the axis of maximum compression*, and *the obtuse angle by the axis of minimum compression which is generally negative, representing tension.*

f) If the position of the principal axes changes from point to point, the shearing surfaces are warped. The less this is the case, i.e., the more nearly homogeneous a material is, the more regular are the shearing planes.

g) The shearing planes do not originate simultaneously, and are not uniformly distributed.

[1] F. Rinne, "Vergleichende Untersuchungen über die Methoden zur Bestimmung der Druckfestigkeit von Gesteinen," *N. Jahrb. f. Min.*, etc., Vol. I (1907), p. 45.

III. HARTMANN'S LAW APPLIED TO EXPERIMENTAL AND FIELD OBSERVATIONS

Hartmann's law enables the geologist as well as the mechanical engineer to reconstruct the position of the principal axes of stress in any given body subject to mechanical deformation—be it a test specimen in the laboratory or the exposed portion of a fractured rock-mass—by analyzing from point to point the position of the planes of shearing. The direction bisecting the acute angle formed by the planes of shearing corresponds to that of the greatest principal axis of compressive stress, while the bisectiix of the obtuse angle gives the direction of the least stress, which in most cases represents active tensile stress. The direction of the intermediate principal stress coincides with the line of intersection of the two planes of shearing.

It is essential, however, to realize at the start the limitations of this law.

a) It applies only to brittle substances.

b) Not all lines of fracture are lines of shearing. Brittle materials, such as cast iron or hard steel, and most rocks under simple tension habitually fail along planes of fracture at right angles to the direction of maximum tensile stress.[1] Soft steel, on the other hand, fails along inclined planes of shearing under tension as well as under compression.

c) The position of the planes must be studied in space, not in any accidental plane of exposure.

d) The principal stresses inferred from them need not be identical with any real stresses, but may be only the resultants of the combined action of several stresses ("equivalent" stresses).

We may now proceed to test the usefulness of Hartmann's law by applying it to a few selected experimental data and geological field observations.

1. *Compressive stress vertical, tensile stress horizontal.—a*) When a cylindrical test piece is subjected to compression beyond the elastic limit, Lüders' lines make their appearance on its surface, forming a characteristic pattern of symmetrical intersecting spiral

[1] See, for instance, A. L. Jenkins, "Combined Stresses," *Jour. Amer. Soc. Mech. Engineers* (1917), p. 696.

lines, with the acute angles formed by their intersection facing the direction of pressure. On specimens of Carrara marble used by Rinne[1] this angle measured 60°, on those used by Kármán[2] it measured 54°, while red sandstone (*Buntsandstein*) gave a value as low as 38°.

When the pressure is increased until rupture occurs, the plane of fracture forms a symmetrical cone with an apical angle equaling the angle of shear characteristic of the material. In this case, the least principal stress equals the intermediate stress. Thereby its position is made indefinite with reference to the infinite number of directions in the plane common to the two lesser stresses, normal to the greatest principal stress. The peculiar conical fracture is the result.[3]

As soon, however, as any one of the infinite number of possible directions in the plane normal to the greatest stress offers a minimum of resistance, rupture occurs[4] along two well-defined planes, as indicated in Figure 3. Daubrée's classical experiments on blocks made of a mixture of plaster of Paris and beeswax[5] correspond directly to this case.

[1] F. Rinne, "Vergleichende Untersuchungen über die Methoden zur Bestimmung der Druckfestigkeit von Gesteinen," *Neues Jahrb. f. Miner.*, etc., Vol. I (1907), p.45.

[2] Th. von Kármán, "Festigkeitsversuche unter allseitigem Druck," *Zeitschr. d. Vereins deutscher Ingenieure*, Vol. LV (1911), pp. 1748–57.

[3] The remarkable fracturing in the form of parallel and interpenetrating cones observed in the brittle white limestones of the Upper Jurassic along the intensely shattered margin of the crypto-volcanic basin of Steinheim seems to be due to this condition. W. Branco u. E. Fraas, "Das kryptovulkanische Becken von Steinheim," *Phys. Abhandl. d. K. Preuss. Akad. d. Wissensch.* (Berlin, 1905), pp. 36–38.

[4] In a cube where four directions offer an identical minimum of resistance, the planes of fracture form a pyramid as may be seen in any ordinary crushing test.

[5] A. Daubrée, "Études synthétiques de géologie expérimentale" (Paris, 1879), pp. 315 ff. and Figs. 93 and 94. For a copy of Fig. 93 see, e.g., Van Hise, "Principles of North American Pre-Cambrian Geology," *Sixteenth Ann. Rep. U.S.G.S.* (1895), Pt. I, p. 644, Fig. 126. Note in this figure the difference between Lüders' lines and the final plane of shearing. The former, marked "R," do not, at first, correspond to continuous internal surfaces. They represent purely local effects along individual lines of stress. The establishment of large planes of shearing (marked "F") is a later development. The difference between the two is shown strikingly on the right side of the block, where the main fracture cuts diagonally across Lüders' lines. This contrast between Lüders' lines and the final planes of rupture is met with in all experiments. It seems to indicate that at first the greatest tension exists parallel to the surface of the test specimen, due to the stretching of the horizontal dimensions accompanying the vertical shortening. Rupture, on the other hand, gives dominance to the direction of easiest movement in a radial direction.

b) It is easy to see that the joints on Mine Fork, Kentucky, described in the introduction to this paper, correspond to this type. Here, however, the active stress was the horizontal tension existing at the top of the anticline, while the weight of the overlying rock-masses, giving the compressive stress, was merely passive.

The analysis of joints can, however, be carried farther and may often yield information of decisive value to the field geologist.

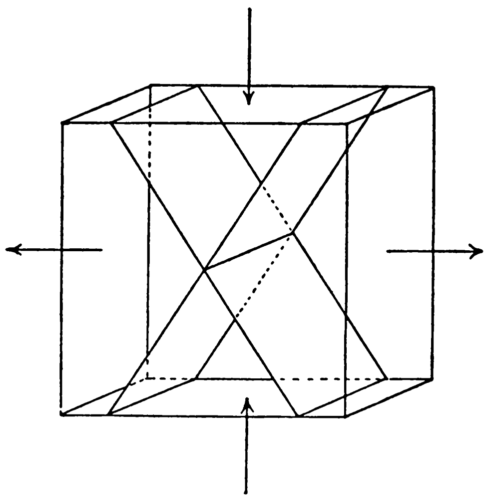


FIG. 3.—Diagram illustrating the position of the planes of shearing in a brittle body subjected simultaneously to vertical compression and horitonzal tension.

A detailed analysis of the joints on Mine Fork will be given here to illustrate the method of analysis used by the writer.

The exposure on Mine Fork is such as not to give the true dip of either of the joint planes. The joints themselves are filled with mineral matter and their surface is nowhere exposed. But their apparent hade was measured on the vertical face of the exposure trending essentially NNE—SSW and their strike was determined on the level top of the cliff.

	Set I	Set II
Apparent hade:	27° northward	35° southward
Strike:	N 78 W	N 27 W

A complete analysis from these data involves the following steps. Find

1. The actual position of the two planes in space.

2. The direction, in space, of the line of intersection of the two planes which corresponds to the position of the intermediate principal stress.

3. The position of the plane normal to this line.

4. The location in this plane of the other two principal stresses bisecting the acute and obtuse angles respectively.

The stereographic projection is admirably adapted to the demands of problems of this kind. By its use, the position of the principal stresses in space can be obtained in the field from any given set of joints within a few minutes. In the following brief description of the construction of Figure 4, a working knowledge of the stereographic projection is assumed.[1]

1. Draw the line *Ex-Ex*, trending N 23 E, to represent the vertical plane of the exposure. On it, mark the point *c*, 27° southward from *O*, and *c′*, 35° northward from *O*. The planes *acb* and *a′c′b′* represent the two joint planes and can now be drawn.

2. Since the two points *O* and *d* are common to both planes, *Od* is the line of intersection of the two planes, that is, the direction of the intermediate stress.

3. On the great circle *acb* mark point *e*, and similarly point *e′* on *a′c′b′* both 90° from *d*. Through *e* and *e′* draw the great circle *fee′g*, representing the plane normal to *Od*. On it we can read directly the true value of the acute angle of the shearing planes, which in this case is 72°.

[1] For a detailed discussion of the stereographic projection see A. Johannsen, *Manual of Petrographic Methods*, p. 17. McGraw-Hill Book Co., 1914. For most purposes a protractor giving great circles and vertical small circles 10° apart, such as is given (after Penfield) in A. F. Rogers, *Introduction to the Study of Minerals* (McGraw-Hill Book Co., N.Y., 1912, pp. 82–86), is perfectly sufficient. It can readily be copied and carried in the notebook for use in the field. Greater accuracy can, of course, be obtained by the use of Wulff's net, a large copy of which is contained in E. E. Wright, "The Methods of Petrographic-Microscopic Research," *Carnegie Inst. Pub. 158*, Pl. III.

The reader who has had little practice in the use of the stereographic projection will find it easy to visualize Fig. 4 by remembering that the great circles must be imagined to be drawn on the surface of a hemisphere resting on the circle NESW with *O* at its center. A line such as *dO*, therefore, represents a radius extending from the surface of the hemisphere, at *d*, downward to the center *O*.

4. Find the point i, located halfway between e and e'. The line iO, lying in a plane normal to the intermediate stress dO, and in the plane *hidj* bisecting the acute angle of the shearing

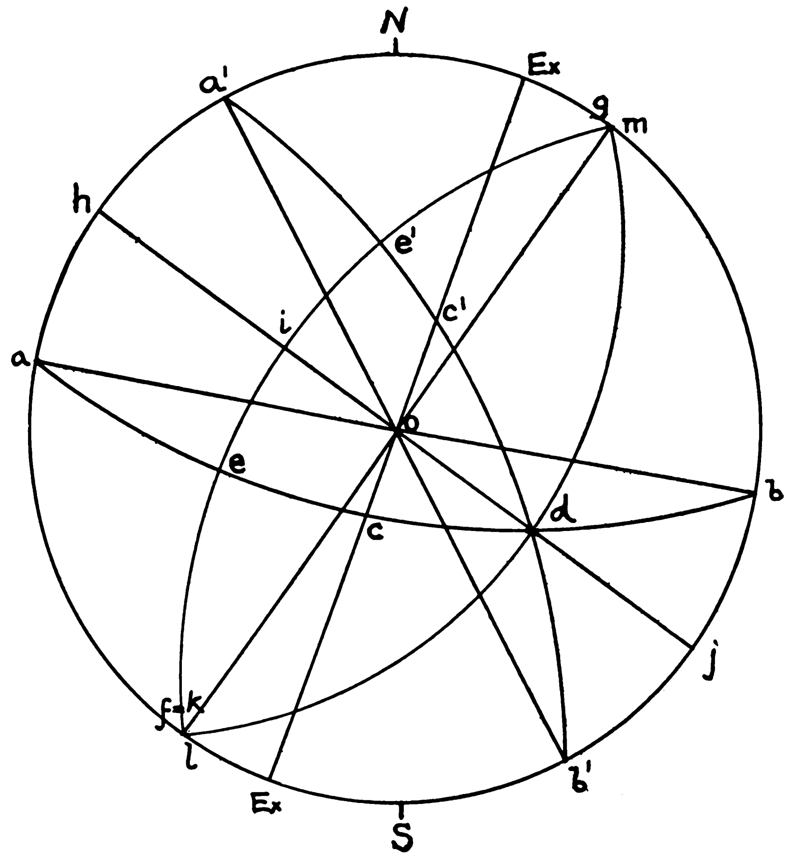


FIG. 4.—Stereographic projection of the joints on Mine Fork, Kentucky. *Ex*= Trend of exposure; *ab*=set I of joints; *a'b'*=set II of joints; *od*=line of intersection of joint planes=position of intermediate principal stress; *hidj*=plane bisecting the acute angle of the joint planes; *oi*=located in this plane, normal to *od*=position of the greatest (compressive) principal stress; *mdl*=plane bisecting the obtuse angle of the joint *hid* planes; *og*=located in the plane *mdl*, normal to *od*=least (tensile) principal stress; *mdl*, *gif*=principal planes.

planes, represents the direction in space of the compressive stress. The line *hj* gives the trend of this stress in a horizontal plane.

5. 90° from i, on the great circle *gif*, mark the point k, which in this case practically coincides with f. The line kO, lying in

the plane normal to *Od* and 90° from *Oi*, represents the direction in space of the tensile stress, and the line *ml*, in the vertical plane *lkdm*, gives the horizontal trend of this stress.

This analysis leads to the following conclusions:

The direction of *Ok*, of the tensile stress, differs only 3° from the horizontal, as would be expected at the crest of an anticlinal bulge.

The pull was slightly inclined downward in the direction N 33 E.

The very crest of the anticline, therefore, must be sought on the left side of the exposure, a short distance to the southwest. The differential movement which develops when strata slip past each other in the process of folding was here directed toward the crest and favored the development of the joints of set I which are more numerous, more regular, and stronger than those of set II.

The direction N 33 E of the greatest tension suggests in a general way the dip. and therewith also the strike, of the strongly cross-bedded sandstone.

2. *Both, compressive and tensile stresses horizontal.*—*a*) When Daubrée subjected to torsion narrow strips of glass, measuring about a yard in length, and produced on them the well-known system of intersecting fractures, he gave the science of geology one of its most impressive laboratory experiments and one of its most popular textbook illustrations on the subject of joints.

Careful analysis, however, reveals the fact that the conjugate systems of fractures which he produced, do not correspond directly to similar joint systems in nature. Figure 5 is a sketch of the fractures forming two prominent "fans" on one of Daubrée's plates.[1]

The tendency to form such "fans" is obvious in all torsion experiments made with glass. Duparc and LeRoyer found that it is the more pronounced, the thicker the glass plate is which is used for the experiment.[2]

[1] The one in the center of the plate reproduced on Plate XII of Haug, *Géologie*, Vol. I (Paris, 1911) (opp. p. 228).

[2] L. Duparc and A. LeRoyer, Contributions à l'étude expérimentale des diaclases produites par torsion," *Archives des Sciences phys. et nat.*, 3me sér. XXII (Genéve, 1889), p. 307. Daubrée used plates 7 mm. thick; on plates of 2 mm. thickness or less, "fans" may not form at all.

Each "fan" consists of a gently curved "master-joint," marked *t* and *t'*, from which start, at a very acute angle, a number of minor joints, *s* and *s'*, which unmistakably tend to be straight and parallel to each other.

The clue to this peculiar fan-structure of fractures we find in Hartmann's experiments with rectangular strips of soft steel.[1] Under torsion, two systems of Lüders' lines appeared on them, each parallel to one of the sides of the test piece, intersecting practically at right angles. This indicates that the direction of greatest tension traverses the surface obliquely, forming an angle near 45° with the axis of torsion.[2] When the deformation was carried farther additional lines of deformation appeared in the vicinity of the longer edges, bisecting the angles formed by the first set of lines.

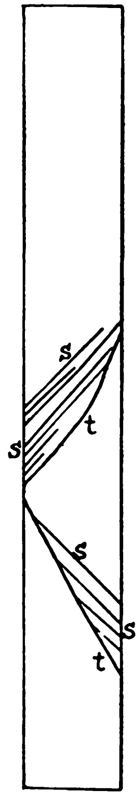


FIG. 5.—The fractures forming two characteristic "fans" on one of the glass plates used in Daubrée's experiments on fractures produced by torsion.

When a plate is subjected to simple torsion, each element of the upper surface suffers simultaneously tension in one direction and compression at right angles to it. The same is true of the lower surface, but with the directions of tension and compression reversed.[3]

At any point on either surface, therefore, the position of the shearing planes is sharply defined through the combined action of tension and compression as shown diagrammatically in Figure 6. In case tension fractures are formed in addition, they bisect the acute angle of the shearing planes. This is what happened in Hartmann's experiment with

[1] Hartmann, *loc. cit.*, p. 175 and Fig. 173.

[2] This can be verified readily by drawing a circle on the flat side of a rubber eraser and twisting it. G. F. Becker, "The Torsional Theory of Joints," *Trans. Amer. Inst. Mech. Engineers*, Vol. XXIV (1895), p. 136.

[3] For the purposes of the following discussion it is important to remember that essentially horizontal tensional stresses arise in surfaces made convex, and similar compressive stresses in surfaces made concave through the process of bending.

strips of soft steel. The second set of fractures, formed after shearing was well under way along Lüders' lines, consisted of tension fractures, one started from the lower, the other from the upper surface.

Glass, on the other hand, being a highly brittle substance, in contrast to soft steel, will fail along tension fractures rather than along planes of shearing. The fractures marked *t* in Figure 5 are the only ones that form when the glass plate used in the experiment is very thin. They must, therefore, be tension cracks, one set produced on the under side, the other, symmetrical to it, on

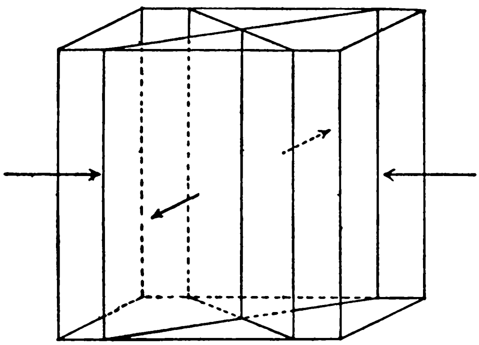


FIG. 6.—Diagram illustrating the position of the planes of shearing in a brittle body subjected simultaneously to compression and tension, both in a horizontal direction.

the upper surface, and both finally extended to both surfaces, owing to the thinness of the plate. The gentle curving of these cracks is quite in harmony with this interpretation.

The other set of fractures, marked *s* in Figure 5, intersects with the tension cracks at angles varying from 15° to 25°. This, however, is one-half of the angle of shearing characteristic of glass.[1]

The same angle for soft steel is approximately 45°. It is evident, therefore, that these fractures represent shearing planes produced by the compressive stress acting in the direction normal to the tensile stress. In the experiments made with glass, however, in contrast to those with mild steel, only one set of the shearing planes forms in connection with a tension crack, that only which

[1] To verify this, it is sufficient to compress small pieces of thick plate glass in a strong vise. The resulting angle of shearing can be measured conveniently by means of Penfield's contact goniometer.

lies favorable to the differential movement resulting from the process of torsion. Here, again, the fractures produced on the upper surface extend down to the lower surface and vice versa.

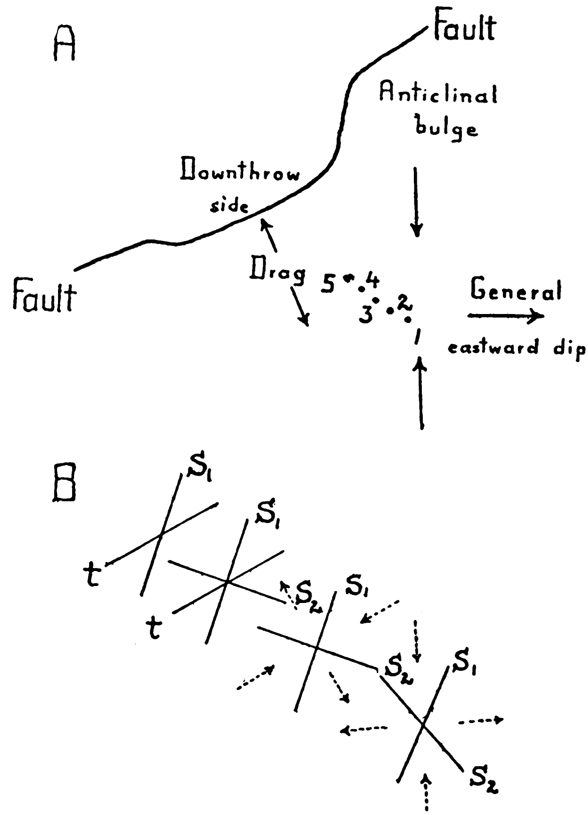


FIG. 7.—*A*. Map sketch, showing relation of stations on Crooked Creek, Adams County, Ohio, at which joints were measured, to the fault and to other structural features. *B*. Diagram showing the position of the joints observed at the stations 1, 2, 3, 4, 5. S_1 S_2=shearing joints; t=tension joints.

Since the intensity of the compressive stress increases with the thickness of the plate, it is evident that the fan-shaped groups of cracks will form the more freely the thicker the plate is.

The most striking feature of Daubrée's famous experiment, therefore, namely the formation of two systems of fractures intersecting approximately at right angles, is an accidental result of the exceptional brittleness of glass and the thinness of the plates

used, permitting the fractures produced on the upper and lower surfaces respectively to interpenetrate.

Most rock materials, on the other hand, are less brittle than glass and therefore more inclined to yield along shearing planes when subjected to torsion. Moreover, at least in the case of the larger joints observed in nature, the thickness of the formations undergoing deformation through torsion is sufficient to keep the fractures formed on the upper and lower surfaces separate.

In general, therefore, joints produced by torsion in the course of larger earth movements should occupy the position indicated in Figure 6 with the direction of both, compressive and tensile stress, not differing much from the horizontal. In addition to these, tension fractures, bisecting the angle of the other joints, may occur and even dominate. These, together with an unequal development of the two sets of shearing joints, with possibly one even missing completely, may give considerable variation to the appearance of the same joint system from point to point.

We may now turn to a discussion of three selected cases of joint systems.

a) In Figure 7*A* the general structural relations are given for five points along Crooked Creek, Adams County, Ohio, at which the position of joints was determined by the writer. The joints here cut in a nearly vertical position through the rather thin and even beds of the fine-grained dolomite of the Bisher formation.

Figure 7*B* shows the strike of these joints as contained in the following field notes.

Station	Set s_1	Set s_2	Set t
1	N 25 E strongly developed	N 40 W sharp and persistent	....
2	N 20 E well developed	N 70 W (average) strong but variable	....
3	N 20 E sharp and regular; closely spaced (2–10 in. apart)	N 70 W (average) few and far apart (several feet); irregular.	....
4	N 22 E sharp and regular; (1–2 feet apart)	N 70 W very few, only three good joints seen	N 62 E sharp and regular, 1–2 feet apart
5	N 20 E sharp and regular	....	N 65 E dominant system, closely spaced

The very regular systems of joints observed at station 1 obviously owe their existence chiefly to the compressive stress caused by the upward buckling of the strata farther north along the fault. The change of the angle of shearing from 65° at station 1, to 90° at stations 2 and 3, probably is due to factors which will be discussed in the second part of this paper. It is brought about by a shifting of the trend of the system s_2 which at the same time becomes more and more irregular and scattered. At station 4 set s_2 is only represented by a few widely separated cracks, while a new system of fractures makes its appearance. They are parallel to the fault and become increasingly prominent and closely spaced as the fault is approached. They must be tension cracks.

The joints at station 5 offer an exact analogy to the fan-shaped cracks formed on the upper surface of a glass plate under torsion, with shearing planes developed only on one side of a tension crack.

The great practical value of such an interpretation of joints, during the progress of field work, is obvious. In this case, for instance, the joints at stations 1 to 3 would lead the field geologist at once to look for an uplift either north or south of these points, or both. The sudden appearance and increasing importance of an additional system such as the joints of set t would suggest the neighborhood of a flexure or a fault not far to the northwest striking N 65 E.

Information such as this will certainly pay for the time spent on detailed intelligent observation.

b) The remarkable jointing exposed along the shores of Lake Cayuga and Lake Seneca. "resembling the gigantic ruins of Cyclopean architecture," has been made classic through the series of woodcuts published by Hall in 1843.[1]

Miss Pearl Sheldon[2] has given us a large number of careful measurements of these joints in the vicinity of Cayuga Lake. Two systems of joints stand out from all others. They are generally stronger, more regular, and remarkably constant in their

[1] *Geology of New York*, Pt. LV (Albany, 1843), pp. 303–6. For good modern illustrations see, e.g., *Watkins Glen-Catatonk Folio*, No. 169 (1909), Pl. I, Figs. 15 and 16; and *Jour. Geol.*, Vol. XX (1912), p. 78.

[2] Pearl Sheldon, "Some Observations and Experiments on Joint Planes," *Jour. Geol.*, Vol. XX (1912), pp. 53–79; 164–83.

trend. They are practically vertical. The strike of the one set ranges from N 70 E to N 80 E with the majority lying between N 75 E to N 78 E. The other set has a strike ranging from N 20 W to N 10 E. Frequently joints of the two extremes, near N 20 W and N 10 E, are present at the same locality.

No detailed data are given for the large number of minor joints of this region. Their trend seems to be highly variable, in general and from point to point, and ranges through all points of the compass. They are often curved and irregular, and as a rule small. They generally show a large hade, ranging as high as 60°.

The remarkably uniform position of the two major joint systems[1] stands in strong contrast to the highly variable dip of the limbs of the low anticlines and synclines formed by the rocks of the region, as shown on the geological map.[2] This contrast is especially striking as Miss Sheldon points out herself, where the dip and strike of the rocks changes rapidly from point to point along the pitching end of an anticline (for instance, the Shurger Point anticline)[3], while the position of the joint planes remains unchanged.

It is evident, therefore, that the formation of these joint systems was independent of the folding and followed it.

Here, as on Crooked Creek, one system is quite constant in its trend, while the other varies in such a way as to form angles ranging from about 65° to 90° with it. We are, therefore, justified in the assumption that they represent planes of shearing produced by compression in a NE–SW direction under general conditions of torsion. To test this interpretation, we turn to the geological maps of Watkins Glen and Catatonk quadrangles.

West of Cayuga Lake, along the axis of the Watkins anticline, the contact of the Portage and Chemung formations is nearly level, varying between 1,480 and 1,560 feet above sea-level for a distance of over 18 miles. As it approaches the valley of Cayuga Inlet, it rises above 1,600 feet. East of Ithaca, in the same general

[1] See especially Figs. 6 and 7 of Miss Sheldon's paper.

[2] See *Watkins Glen-Catatonk Folio*, No. 169.

[3] *Loc. cit.*, p. 67.

direction, the contact rises rapidly to over 2,100 feet in a similar distance. A closer inspection of the northeast corner of Catatonk Quadrangle reveals the presence of considerable doming in the general direction suggested by the position of the joint planes. This broad anticlinal bulge does not seem to be mentioned in the text of the folio. The fact that the writer's attention was called to it through the analysis of the joints of Cayuga Lake, serves well to illustrate the practical possibilities of the method employed.

The presence of an uplift to the northeast accounts for the existence of a compressive stress in that direction. The horizontal tensile stress implied by the position of the joint planes can be accounted for equally well. Crossing the shores of Cayuga Lake in a southeasterly direction (suggested by the obtuse angle of the joints), we find that, on the crests of the Fir Tree Point, Watkins and Alpine anticlines, the contact of the Portage and Chemung formations remains essentially at an elevation between 1,600 and 1,700 feet above sea-level. Beyond the Alpine anticline, however, in the same southeasterly direction, within a similar distance, the same contact drops to near 1,000 feet in the vicinity of Jenksville in Newark Valley Township.

The existence of this depression in the direction suggested by the position of the joint planes, leaves little doubt that this relatively pronounced flexure gave rise to the tensile stress involved in the formation of the joints.

c) For a last example we turn to Thwaites's paper on the "Sandstones of the Wisconsin Coast of Lake Superior."[1]

When we plot the strike of the joints of this region as recorded in the table on page 96, it appears that the peninsula north of Washburn, including the Apostle Islands, in contrast to the regions to the west and south, is traversed by two dominant and persistent systems of major joints. One of the two strikes on the average E–W, the other about 10° east of north. Most probably they represent planes of shearing. The position of the acute angle points to the action of a compressive stress in a NE–SW direction, with a tensile stress acting in a NW–SE direction.

[1] F. T. Thwaites, "Sandstones of the Wisconsin Coast of Lake Superior," *Wis. Geol. and Nat. Hist. Sur., Bull. 25* (1912).

The map accompanying Thwaites's paper shows that the jointed area lies in the continuation of the northeastern end of the great Douglas thrust fault. If this fault had any horizontal component in the northeasterly direction, it could have supplied the compressive stress responsible for the position of the joints.

Unfortunately, the fault contact of the Middle Keweenawan traps with the underlying much younger Orienta sandstone was found exposed only at four localities. At three of these, the exposures were not even found sufficient to measure the hade of the thrust plane.[1]

In the vicinity of the falls of the Amnicon River, however, the fault-plane proper was found exposed at two separate localities, about 500 feet apart. Here, at both points, two systems of grooves were observed on slickensided surfaces in the immediate vicinity of the fault, on surfaces of conglomeratic beds of sandstone which represent most probably shreds of lower beds dragged up along the thrust-plane.[2] One of the sets of grooves "is parallel to the dip, the other is inclined at an angle of about 30° in a NE–SW direction."[3]

Although the grooves are not part of the fault-planes proper, but occur on what seem to be irregular fragments of sandstone wedged in front of the fault, their constancy on seemingly different planes at points 500 feet apart can hardly be looked on as due to purely local movements. They are more likely the direct result of the last movements along the major thrust-plane and essentially parallel to them.

If the joints in the vicinity of the Apostle Islands really owe their origin to the action of this pressure directed upward at an angle of about 30° toward the northeast, we should find evidence of it in the position of the joint planes themselves. According to the table on page 96 of Thwaites's paper the joints have the tendency to be vertical. From the text we learn, however, in addition that "many of the E–W joints are inclined, usually at a steep angle to the north."[4]

[1] F. T. Thwaites, *op. cit.*, pp. 66, 76, 80, 81. [2] *Ibid.*, p. 83.

[3] *Ibid.*, p. 78. [4] *Ibid.*, p. 94.

When we plot the attitude of these joints, using the stereographic projection as explained on page 10 of this paper, we find that the greatest (compressive) principal stress should have been directed upward from the southwest at an angle of 15° if we assume the E–W joints to dip 70° north, and of 30°, if they dip 50° north.

This correspondence is quite striking and leaves little doubt as to the correctness of this interpretation.

The northwest-southeast tension, indicated by the position of the obtuse angle of the joints in the Apostle Islands, is parallel to the gentle dip of the rocks of the Bayfield Group. Both dip and tension probably resulted from settling along the axis of the Lake Superior syncline simultaneous with the last movements along the thrust-plane.

d) The last two examples serve well to show how important it is that each observation of jointing be studied in its geographical and structural relations to all others. To assemble the joints observed over a large area in a single diagram means to veil their true relationships. A diagram, published by Hobbs in 1905,[1] shows the strike of 1,004 joints measured by Mr. C. G. Brown in the vicinity of Cayuga and Seneca lakes, New York. It clearly indicates the existence of two nearly orthogonal double systems of conjugate joints. A comparison with Miss Sheldon's diagram[2] shows that the earlier diagram represents a composite picture of the joint systems of two different localities, since the vicinity of Cayuga Lake exhibits only one of the two double systems, as described above.

A fine example of carefully recorded data giving definite measurements of strike and dip, and accurate geographical location of every joint measured, may be seen, for instance, in R. S. Tarr's field observations embodied in Shaler's "Geology of Cape Ann."[3]

3. *Greatest compression horizontal, least compression vertical.*—The position of the principal stresses and of the resulting planes

[1] W. H. Hobbs, "Examples of Joint-controlled Drainage from Wisconsin and New York," *Jour. Geol.*, Vol. XIII (1905), p. 370.

[2] *Loc. cit.*, p. 66.

[3] N. S. Shaler, "The Geology of Cape Ann, Massachusetts," *Ninth Ann. Rep.*, *U.S.G.S.* (1889), pp. 597–602 and Pls. 72–74.

of shearing for this grouping of stresses is represented diagrammatically in Figure 8; Figure 9 illustrates the occurrence of this type in nature on a small scale.[1] It shows "symmetrical faults" in a hard encrinal layer a foot or two in thickness in the Hamilton shales, exposed along the shores of Cayuga Lake. "The exposures of this layer along the lake show faults every few feet." "The strike of the majority is from 20–25° north of west." "Their inclination is sometimes south and sometimes north and the angles are nearly

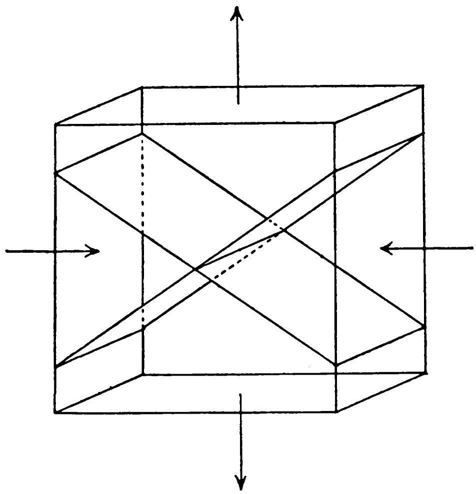


FIG. 8.—Diagram illustrating the position of the planes of shearing in a brittle body subjected to compression in a horizontal direction with the direction of easiest relief (least principal stress) vertical.

the same in the two cases, making the faults symmetrical about a nearly horizontal plane." "The hade varies from 45° to 75°, but most are near the average, which is 62°." The fault surfaces are slickensided and covered with strong, even striations. "The vertical displacement along these faults is from a fraction of an inch to three inches." The faults "usually continue for a few feet in the adjacent shale, but instead of continuing with the same hade, they flatten out and become nearly horizontal in the shales where no hard layer is present."[2]

[1] Pearl Sheldon, "Some Observations and Experiments on Joint Planes," *Jour. Geol.*, Vol. XX (1912), Fig. 3, p. 61.

[2] *Ibid.*, pp. 60–62.

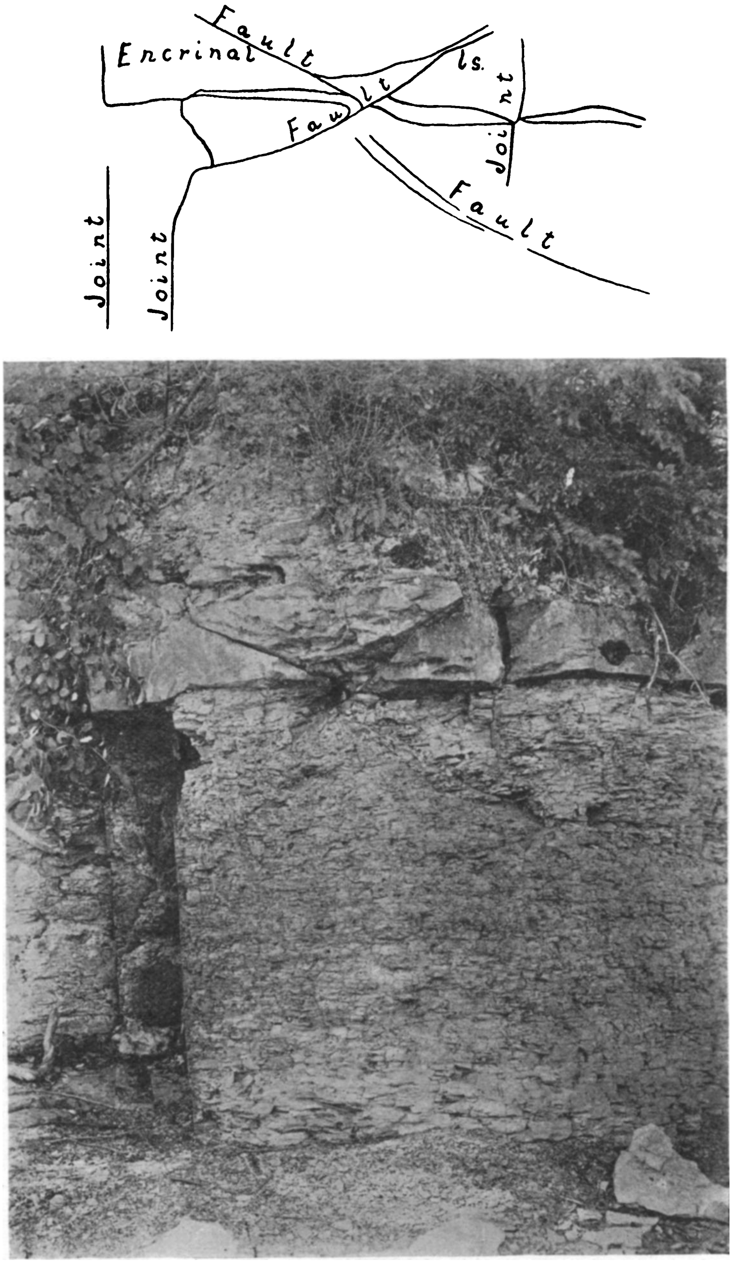


FIG. 9.—Planes of shearing due to stress relations similar to those illustrated in Fig. 9. Bed of hard encrinal limestone in Hamilton shales, exposed along shore of Cayuga Lake. (P. Sheldon, 1912.)

Horizontal faults "occur by the score in the shale beds," usually offsetting the vertical joints for distances often measuring several inches.

These "faults" are unquestionably shearing planes forming an acute angle of approximately 60° facing the direction of the horizontal compressive stress which is clearly manifested in the differential movement between adjoining layers of shale referred to above as "horizontal faults."

Identical results have been obtained in the laboratory, whenever sufficiently brittle materials were subjected to horizontal compression in the course of experiments on overthrusting,[1] and before our eye loom up the sections of the mountain ranges which these experiments were to help explain, with thrust faults in astonishing numbers and on gigantic scales.

Here the subject of our discussion assumes different proportions. We realize that the grouping of stresses resulting in the formation of any of the fracture systems discussed before remains the same whether the fracturing finally results in the formation of vast lines of displacement generally referred to as major faults, which may bound mountain ranges or even continents, or ends with the production of minute cracks which, filled with white calcite, form a delicate network on the dark rock and, found on the surface of flat pebbles on the wet beach, are the delight of children.

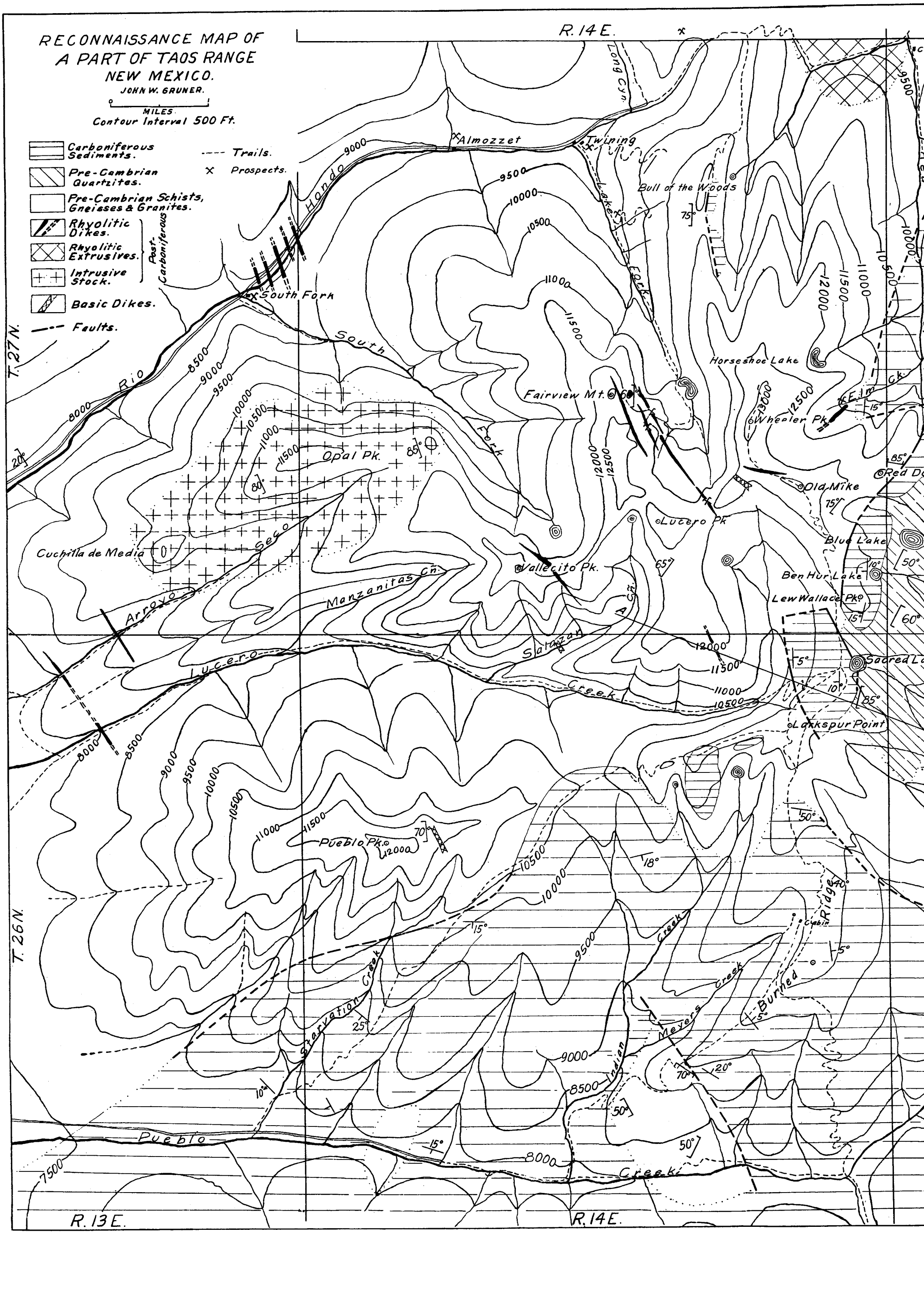
Before we can extend the application of Hartmann's law to the larger scale of the great overthrusts of folded mountains, we must first answer a question which now assumes fundamental importance: Is the angle of shear, in a given substance, sufficiently constant under widely different conditions of pressure and temperature so as to exclude the possibility of grave errors in its use?

We may approach this question best by turning to the ingenious mathematical theory which Mohr has given to account for the results of Hartmann's and others' experiments.

[1] See, for instance, H. M. Cadell, "Experimental Researches in Mountain Building," *Trans. Roy. Soc. Edinburgh*, Vol. XXXV (1890), pp. 337–57; and R. T. Chamberlin and W. Z. Miller, "Low-Angle Faulting," *Jour. Geol.*, Vol. XXVI (1918), pp. 1–44, especially Fig. 9.

Note, however, that the position of the strain ellipsoids in the rigid layers in Fig. 10 does not correspond to the writer's interpretation.

[*To be continued*]


RECONNAISSANCE MAP OF
A PART OF TAOS RANGE
NEW MEXICO.
JOHN W. GRUNER.
MILES
Contour Interval 500 Ft.
Carboniferous Sediments.
Pre-Cambrian Quartzites.
Pre-Cambrian Schists, Gneisses & Granites.
Rhyolitic Dikes.
Rhyolitic Extrusives.
Post-Carboniferous
Intrusive Stock.
Basic Dikes.
Faults.
Trails.
Prospects.
R.14E.
R.13E.
R.14E.
T.27N.
T.26N.
Almozzet
Twining
Long Cyn.
Bull of the Woods
Lake Fork
Hondo
Rio
South Fork
South Fork
Horseshoe Lake
Fairview Mt.
Wheeler Pk.
Red Dome
Old Mike
Opal Pk.
Lucero Pk.
Blue Lake
Cuchilla de Media
Seco
Arroyo
Vallecito Pk.
Manzanitas Cn.
Ben Hur Lake
Lew Wallace Pk.
Salazar
Lucero
Creek
Sacred Lake
Larkspur Point
Pueblo Pk.
Ridge
Cabin
Creek
Starvation Creek
Meyers Creek
Burned
Indian
Pueblo
Creek
7500
8000
8500
9000
9500
10000
10500
11000
11500
12000
12500
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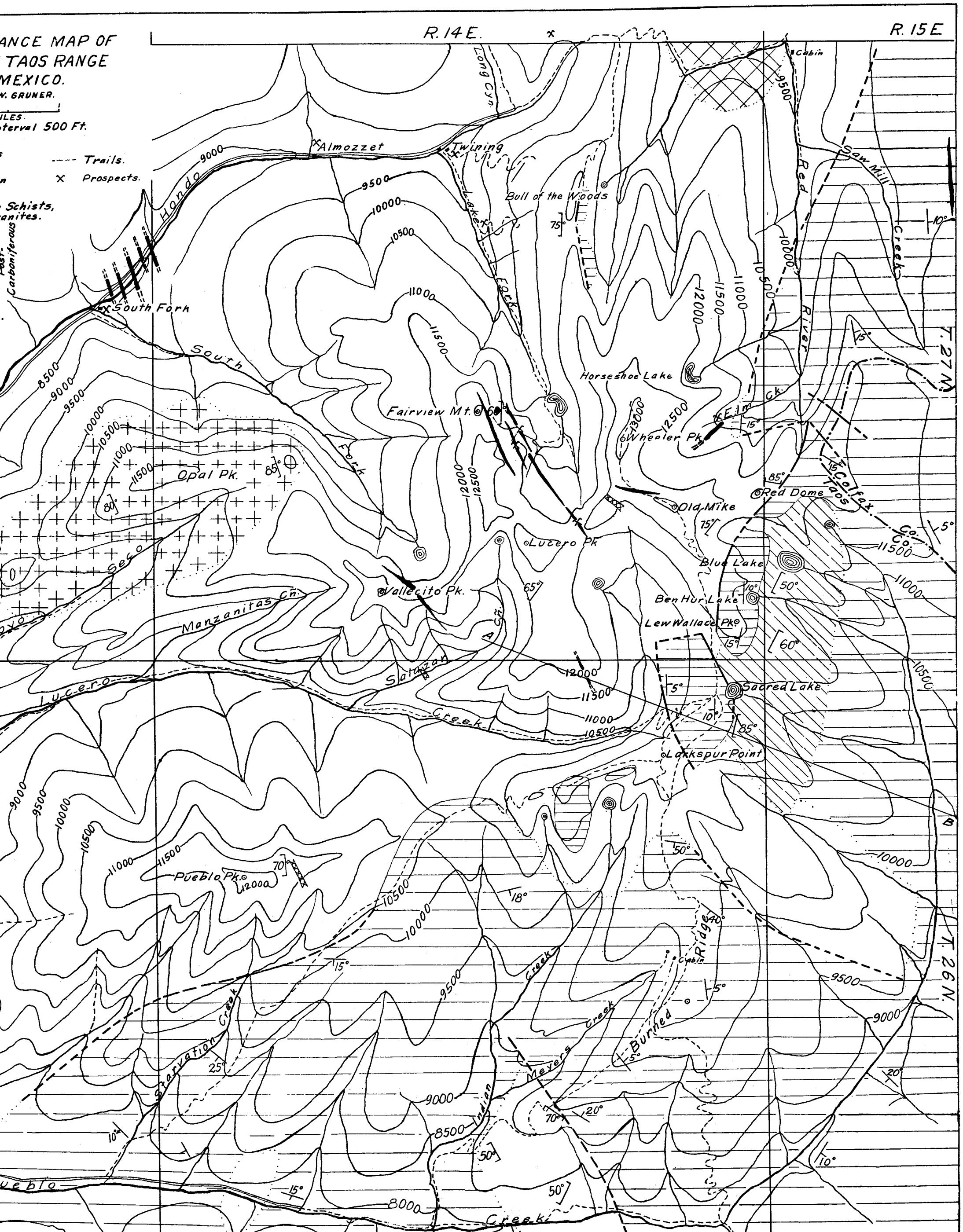
ANCE MAP OF
TAOS RANGE
MEXICO.
V. GRUNER.
ILES.
terval 500 Ft.
---- Trails.
× Prospects.
Schists,
anites.
Carboniferous
R.14E.
R.15E.
Long Cyn.
Almozzet
Twining
Hondo
Lake Fork
Bull of the Woods
Red River
Saw Mill Creek
South Fork
South Fork
Horseshoe Lake
Fairview Mt.
Wheeler Pk.
E. Im Ck.
Opal Pk.
Old Mike
Red Dome
Colfax Co.
Taos Co.
Lucero Pk.
Blue Lake
Vallecito Pk.
Seco
Manzanitas Cn.
Ben Hur Lake
Lew Wallace Pk.
Salazar Cn.
Lucero
Creek
Sacred Lake
Larkspur Point
T. 27 N.
T. 26 N.
Pueblo Pk.
Ridge
Cabin
Burned
Creek
Starvation Creek
Meyers Creek
Indian
Pueblo
Creek
R.14E.
R.15E.